GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2018

MARKING GUIDELINES

10611	MATHEMATICS (PAPER 1)

14 pages

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

MATHEMATICS
(Paper 1)

MARKING GUIDELINES

	QUESTION 1	
1.1		
1.1.1	$x^2 - x - 30 = 0$ $(x-6)(x+5) = 0$ $x = 6$ or $x = -5$	✓correct factors ✓both answers correct (2)
1.1.2	$3x^2 - 8x - 4 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-4)}}{2(3)}$ $x = 3{,}10$ or $x = -0{,}43$	✓standard form ✓correct subst. in the correct formula ✓ $x = 3{,}10$ ✓ $x = -0{,}43$ **Penalise for incorrect rounding off here only** (4)
1.1.3	$\sqrt{5-x} - x = 1$ $\sqrt{5-x} = 1 + x$ $(\sqrt{5-x})^2 = (1+x)^2$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4$ or $x = 1$ NA	✓isolate square root ✓square both sides ✓standard form ✓both answers correct ✓choose correct answer (5)
1.1.4	$\frac{6x^2 - 3x}{3} \leq 3x^2$ $2x^2 - x \leq 3x^2$ $2x^2 - 3x^2 - x \leq 0$ $-x^2 - x \leq 0$ $-x(x+1) \leq 0$ $x \geq 0$ or $x \leq -1$ **OR**	✓simplify ✓standard form ✓correct factors ✓critical values ✓correct inequalities

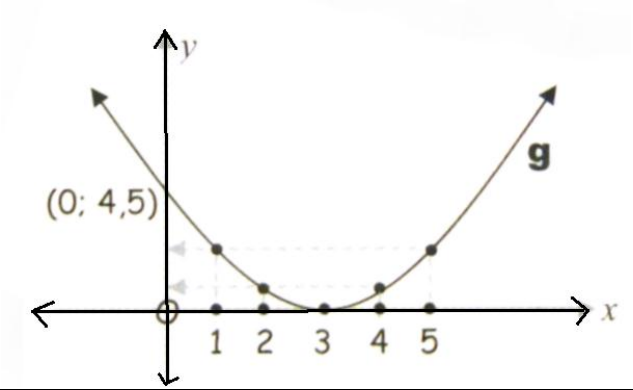
	$\frac{6x^2-3x}{3} \le 3x^2$ $6x^2-3x \le 9x^2$ $6x^2-9x^2-3x \le 0$ $-3x^2-3x \le 0$ $3x(x+1) \ge 0$ $x \ge 0$ or $x \le -1$	✓simplify ✓standard form ✓correct factors ✓critical values ✓correct inequalities (5)
1.1.5	$2x^2+7.\sqrt{2x}=2$ $2^2.2^x+7.2^{\frac{x}{2}}-2=0$ $4.2^x+7.2^{\frac{x}{2}}-2=0$ $(4.2^{\frac{x}{2}}-1)(2^{\frac{x}{2}}+2)=0$ $2^{\frac{x}{2}}=\frac{1}{4}$ or $2^{\frac{x}{2}}=-2$ $2^{\frac{x}{2}}=2^{-2}$ NA $\frac{x}{2}=-2$ $x=-4$	✓standard form ✓factors ✓ $2^{\frac{x}{2}}=2^{-2}$ ✓NA ✓x = -4 (5)
1.2	$6x^2+2px-3x-p=0$ $6x^2+x(2p-3)-p=0$ $\Delta=b^2-4ac$ $\Delta=(2p-3)^2-4(6)(-p)$ $=4p^2+12p+9$ $=(2p+3)^2$ Δ = perfect square $\therefore$ rational roots	✓coefficient of x ✓correct substitution in correct formula ✓factors ✓$\therefore \Delta$ = perfect square (conclusion) (4)
		[25]

	QUESTION 2	
2.1	$T_n = a + (n-1)d$ $-239 = 6 + (n-1)(-5)$ $-239 = 6 - 5n + 5$ $250 = 5n$ $n = 50$	✓T_n =–239 ✓d = –5 ✓answer (3)
2.2	$T_3 = ar^2 = 18$ ...① $T_5 = ar^4 = 162$ ...② ② ÷ ①: $r^2 = 9$ (r < 0) $r = -3$ $ar^2 = 18$ $a(9) = 18$ $a = 2$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_7 = \frac{2((-3)^7 - 1)}{-3 - 1}$ $= \frac{2(-2187 - 1)}{-4}$ $= 1094$	✓eq① ✓eq② ✓r = -3 ✓a = 2 ✓correct substitution in correct formula ✓answer (6)
2.3	3; x; 11; 21; 35 $x - 3$; $11 - x$; 10; 14 $-2x = 4 - 14$ $-2x = -10$ $x = 5$	✓first difference d_1 ✓equating second difference ✓ answer (3)

2.4	$S_8 : S_4 = 97:81$ $\frac{S_8}{S_4} = \frac{97}{81}$ $\frac{a(1-r^8)}{1-r} \div \frac{a(1-r^4)}{1-r} = \frac{97}{81}$ $\frac{a(1-r^8)}{1-r} \times \frac{1-r}{a(1-r^4)} = \frac{97}{81}$ $\frac{(1-r^8)}{(1-r^4)} = \frac{97}{81}$ $\frac{(1-r^4)(1+r^4)}{(1-r^4)} = \frac{97}{81}$ $1+r^4 = \frac{97}{81}$ $r^4 = \frac{16}{81}$ $r^4 = \left(\frac{2}{3}\right)^4$ $r = \left(\frac{2}{3}\right)$ $\therefore\ r = \left(\frac{2}{3}\right)$ First three terms: 9; 6; 4	✓ $\frac{a(1-r^8)}{1-r} \div \frac{a(1-r^4)}{1-r} = \frac{97}{81}$ ✓simplify $\frac{(1-r^8)}{(1-r^4)} = \frac{97}{81}$ ✓factors ✓ $r^4 = \left(\frac{2}{3}\right)^4$ ✓ $r = \left(\frac{2}{3}\right)$ ✓first three terms (6)
2.5.1	$2(p-5)+2(p-5)^2+2(p-5)^3+...$ $r = p-5$ Convergent: $-1 < r < 1$ $-1 < p-5 < 1$ $4 < p < 6$	✓ $r = p-5$ ✓ $-1 < p-5 < 1$ ✓ $4 < p < 6$ (3)
2.5.2	$S_\infty = \frac{a}{1-r}$ $= \frac{-1}{1-(-\frac{1}{2})}$ $= -\frac{2}{3}$	✓substitute $a = -1$ ✓ $r = -\frac{1}{2}$ ✓ answer (3)
		[24]

	QUESTION 3	
3.1	$A = P(1-i)^n$ $100\,000 = 134\,000(1-0{,}068)^n$ $\frac{50}{67} = (0{,}932)^n$ $n = \log_{0{,}932}\left(\frac{50}{67}\right)$ $n = 4{,}16$ *years*	✓correct formula ✓correct substitution ✓ $n = \log_{0{,}932}\left(\frac{50}{67}\right)$ ✓answer (4)
3.2.1	$A = P(1+i)^n$ $A = 150\,000\left(1+\frac{0{,}1525}{12}\right)^2$ $= R153\,836{,}73$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $153\,836{,}73 = \frac{x\left[1-\left(1+\frac{0{,}1525}{12}\right)^{-24}\right]}{\frac{0{,}1525}{12}}$ $x = R7\,477{,}31$ $\therefore$ The monthly payments are $R7\,477{,}31$ per month	✓ $n = 2$ ✓ $i = \frac{0{,}1525}{12}$ ✓correct formula and substitute for P ✓ $n = -24$ ✓answer (5)
3.2.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{7\,477{,}31\left[1-\left(1+\frac{0{,}1525}{12}\right)^{-6}\right]}{\frac{0{,}1525}{12}}$ $= R42\,934{,}09$ **OR**	✓substitute in correct formula ✓ $i = \frac{0{,}1525}{12}$ ✓ $n = -6$ ✓answer

	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $F = \frac{7\,477{,}31\left[\left(1+\frac{0{,}1525}{12}\right)^{18} - 1\right]}{\frac{0{,}1525}{12}}$ $F = R150\,164{,}35$ $A = P(1+i)^n$ $A = 153\,836{,}73\left(1+\frac{0{,}1525}{12}\right)^{18}$ $A = R193\,098{,}52$ $\therefore$ The outstanding balance is R42 934,17	✓ $n = -18$ ✓ $i = \frac{0{,}1525}{12}$ ✓ $153\,836{,}73\left(1+\frac{0{,}1525}{12}\right)^{18}$ ✓answer (4)
		[13]
	QUESTION 4	
4.1	$y = -8$	✓answer (1)
4.2	$f(x) = 0$ $2^x - 8 = 0$ At P : $2^x = 8$ $2^x = 2^3$ $x = 3$ P(3 ; 0) $x = 0$ At D: $f(x) = 2^0 - 8$ $= 1 - 8$ $= -7$ D(0 ; –7)	✓$2^x = 2^3$ ✓P(3;0) ✓$2^0 = 1$ ✓answer D(0; – 7) (4)
4.3	$h(x) = f(2x) + 8$ $= 2^{2x} - 8 + 8$ $= 2^{2x}$ $h(x) = 2^{2x}$ **OR** $h(x) = 4^x$	✓substitution ✓answer (2)
4.4	$y = 4^x$ $h^{-1} : x = 4^y$ $y = \log_4 x$ **OR**	✓interchange x and y ✓answer

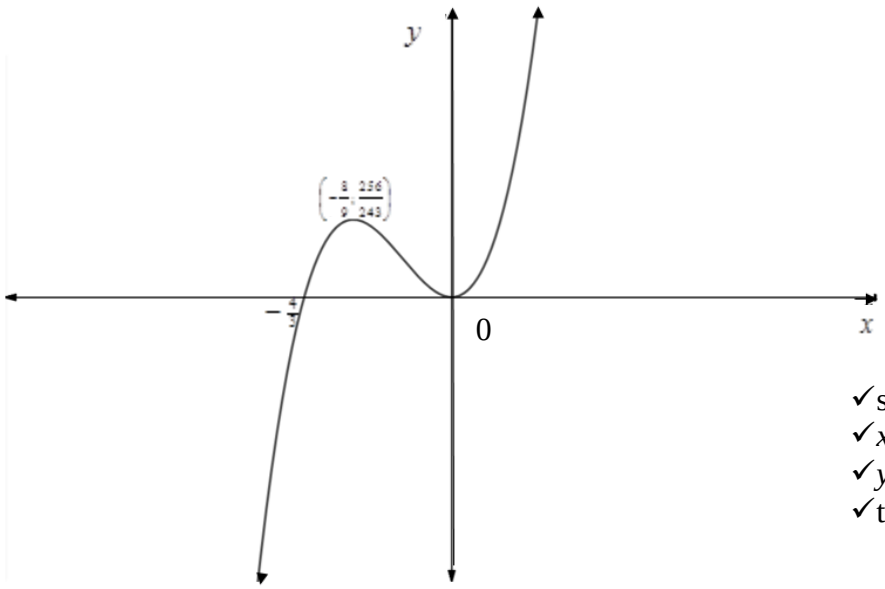
	$x = 2^{2y}$ $2y = \log_2 x$ $y = \frac{1}{2}\log_2 x$ $\log_2 \sqrt{x}$	✓ interchange x and y ✓answer (2)
4.5	$y \in R$ **OR** $y \in (-\infty;\ \infty)$	✓ answer (1)
4.6	$y = a(x-p)^2 + q$ $y = a(x-3)^2$ $4{,}5 = a(0-3)^2$ $\frac{1}{2} = a$ $y = \frac{1}{2}(x^2 - 6x + 9)$ **OR** $y = \frac{1}{2}(x-3)^2$ **OR** $y = \frac{1}{2}x^2 - 3x + 4{,}5$	✓substitution of points D and P ✓value of a ✓equation (in any form) (3)
4.7	By symmetry: $g(1) = g(5)$ *and* $g(2) = g(4)$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= [g(0) + g(1) + g(2) + g(3)] - [g(4) + g(5)]$ $= g(0) + [g(1) - g(5)] + [g(2) - g(4)] + g(3)$ $= 4{,}5 + 0 + 0 + 0$ $= 4{,}5$  	✓correct deduction ✓expansion ✓answer (3)

4.8	Any upwards translation will result in non-real roots.	✓answer (1)
		[17]
	QUESTION 5	
5.1	$x = -2$ or $x = 4$	✓$x = -2$ ✓$x = 4$ (2)
5.2	$x < -2$ or $-2 < x \leq 6$ **OR** $x \leq 6 ; x \neq -2$	✓ $x < -2$ ✓ $-2 < x \leq 6$ **OR** ✓ $x \leq 6$ ✓ $x \neq -2$ (2)
5.3	$-2 \leq x \leq 2$	✓correct critical values ✓correct inequalities (2)
		[6]
	QUESTION 6	
6.1	$g(x) = \frac{1}{4}x^2$	✓answer (1)
6.2	$h(x) = \frac{1}{4}(x)^2 - 2$	✓answer (1)
6.3	$y \geq -2$ **OR** $y \in [-2 ; \infty)$	✓answer (correct inequality / bracket) (1)
		[3]
	QUESTION 7	
7.1	$x = 2$	✓answer (1)
7.2	Translation of 2 units to the right and 3 units downwards.	✓2 units right ✓3 units down (2)
7.3	$x \in R; x \neq 3$	✓answer (1)
7.4	$y = x + c$ $0 = 2 + c$ $-2 = c$ $y = x - 2$	✓correct gradient (m=1) ✓substitution C(2 ; 0) ✓equation (3)
		[7]

	QUESTION 8	
8.1	$f(x+h)=1-3(x+h)^2$ $=1-3(x^2+2xh+h^2)$ $=1-3x^2-6xh-3h^2$	✓ $f(x+h)=$ $1-3x^2-6xh-3h^2$
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{1-3x^2-6xh-3h^2-\left(1-3x^2\right)}{h}$ $=\lim\limits_{h\to 0}\dfrac{1-3x^2-6xh-3h^2-1+3x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-6xh-3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-6x-3h)}{h}$ $=-6x$	✓correct substitution into correct formula ✓simplify $\left(-6xh-3h^2\right)$ ✓common factor h ✓ $-6x$ (5)
8.2	$f'(2)=-6(2)$ $=-12$	✓substitute $x=2$ ✓ answer (2)
		[7]
	QUESTION 9	
	In this question, penalise learners ONCE for incorrect notation.	
9.1.	$\dfrac{d}{dt}\left[(t-2)(t+3)\right]$ $=\dfrac{d}{dt}\left[t^2+t-6\right]$ $=2t+1$	✓correct quadratic expression ✓ $2t$ ✓ 1 (3)
9.2	$D_x\left[\dfrac{5x^3-4}{x}\right]$ $=D_x\left[5x^2-\dfrac{4}{x}\right]$ $=D_x\left[5x^2-4x^{-1}\right]$ $=10x+4x^{-2}$	✓simplification ✓ $10x$ ✓ $4x^{-2}$ (3)
		[6]

	QUESTION 10	
10.1	$f(x) = ax^3 + bx^2$ $7 = a + b$.....(1) $f'(x) = 3ax^2 + 2bx$ $17 = 3a(1)^2 + 6(1)$ $17 = 3a + 2b$.....(2) from (1) $b = 7 - a$ Substitute in (2) $3a + 2(7 - a) = 17$ $3a + 14 - 2a = 17$ $a = 3$ $b = 4$ **OR** $f(x) = ax^3 + bx^2$ $7 = a + b$ $f'(x) = 3ax^2 + 2bx$ $17 = 3a(1)^2 + 6(1)$ $17 = 3a + 2b$ from (1) $a = 7 - b$ Substitute in (2) $3(7 - b) + 2b = 17$ $21 - 3b + 2b = 17$ $-b = -4$ $b = 4$ $a = 3$	✓ $7 = a + b$ ✓ derivative ✓ $17 = 3a + 2b$ ✓ $b = 7 - a$ ✓ $a = 3$ (must show calculation) ✓ $b = 4$ (must show calculation) ✓ $7 = a + b$ ✓ derivative ✓ $17 = 3a + 2b$ ✓ $a = 7 - b$ ✓ $b = 4$ (must show calculation) ✓ $a = 3$ (must show calculation) (6)
10.2	$f(x) = 3x^3 + 4x^2$ $f'(x) = 9x^2 + 8x$ $9x^2 + 8x = 17$ $9x^2 + 8x - 17 = 0$ $(9x + 17)(x - 1) = 0$ $x = -\frac{17}{9}$ or $x = 1$ $y = 3\left(-\frac{17}{9}\right)^3 + 4\left(-\frac{17}{9}\right)^2$ $= -5{,}95$ The other point is $(-1{,}89; -5{,}95)$	✓ derivative ✓ equate to 17 ✓ standard form ✓ factors ✓ x-value ✓ y-value (6)

10.3



✓shape
✓x-intercepts
✓y-intercept
✓turning points

OR

$x-$intercepts

$0=3x^3+4x^2$

$0=x^2(3x+4)$

$x=0 \quad \text{or} \quad x=-\frac{4}{3}$ ✓x-intercepts

$y-$intercepts: $(0;0)$ ✓y-intercepts

Turning point

$0=9x^2+8x$

$x(9x+8)=0$

$x=0 \quad \text{or} \quad x=-\frac{8}{9}$

$y=3(-\frac{8}{9})+4(-\frac{8}{9})^2$

$=\frac{256}{243}$ or 1,05

$(0;0)$ or $(-\frac{8}{9};\frac{256}{243})$ ✓turning points

(If no sketch – award 3 marks for correct calculations) (4)

10.4

$f(x)=3x^3+4x^2$

$f'(x)=9x^2+8x$

$f''(x)=18x+8$ ✓ $f''(x)=18x+8$

$18x+8>0$ ✓$18x+8>0$

$x>-\frac{8}{18}$ ✓answer

OR

$x>-\frac{4}{9}$ (3)

[19]

	QUESTION 11	
11.1	$s(t) = 6000 - 600t - 0{,}2t^3 + 2\times10^{-3}t^5$ $s'(t) = -600 - 0{,}6t^2 + 10^{-2}t^4$ $s'(0) = -600 - 0{,}6(0)^2 + 10^{-2}(0)^4$ $s'(0) = -600$ Thus meteorite is approaching at $600\,m/s$	✓derivative ✓substitute with 0 ✓answer (600 m/s) (3)
11.2	$s(t) = 6000 - 600t - 0{,}2t^3 + 2\times10^{-3}t^5$ $s(10) = 6000 - 600(10) - 0{,}2(10)^3 + 2\times10^{-3}(10)^5$ $= 0$ The meteor will be 0 m away from the planet, thus hitting it.	✓substitution ✓answer (2)
11.3	$s''(t) = -1{,}2t + 4\times10^{-2}t^3$ $s''(5) = -1{,}2(5) + 4\times10^{-2}(5)^3$ $= -1\,\text{m/s}^2$	✓second derivative ✓substitution ✓answer (3)
		[8]
	QUESTION 12	
12.1	No. $P(A \cap B) \neq 0$ **OR** $P(A \cap B) = 0{,}1596$	✓No ✓ $P(A \cap B) \neq 0$ **OR** ✓ $P(A \cap B) = 0{,}1596$ (2)
12.2	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0{,}38 + 0{,}42 - 0{,}16$ $= 0{,}64$	✓substitution ✓answer (2)
12.3	$n(S) = \dfrac{456}{0{,}38}$ $= 1\,200$	✓substitution ✓answer (2)
12.4	$n(C') = 1\,200(0{,}72)$ $= 864$	✓substitution ✓answer (2)
		[8]

	QUESTION 13	
13.1	$P(E) = \frac{n(E)}{n(S)}$ $= \frac{3!}{5!}$ $= \frac{1}{20}$ $= 0,05$	✓3! ✓5! ✓answer (3)
13.2	Children 7! Grandparents 2! Parents 2! × 2! × 2! ×3! = 48 Number of arrangements 7! × 2! × 48 = 483 840	✓7! ✓2! ✓48 ✓answer (4)
		[7]
		TOTAL: 150